# Written Exam Economics summer 2016 

## Microeconomics III

Date: 15 June 2016
(2-hour closed book exam)

Please note that the language used in your exam paper must be English.

This exam consists of 5 pages in total

## PLEASE ANSWER ALL QUESTIONS. PLEASE EXPLAIN YOUR ANSWERS.

1. Consider the following game $G$.

Player 2

Player 1

|  | L | C | R |
| :---: | :---: | :---: | :---: |
| U | 6,3 | 3,0 | 0,1 |
| M | 5,11 | 1,7 | 10,10 |
| D | 2,5 | 2,7 | 0,0 |
|  |  |  |  |

(a) Solve the game by iterated elimination of strictly dominated strategies. If you get a unique solution, indicate this. If your solution is not unique, write up the reduced game where you have eliminated the strictly dominated strategies.
(b) Find all the pure and mixed-strategy Nash Equilibria.
(c) What are the Subgame-perfect Nash Equilibria of the game $G(2)$, i.e. the game $G$ repeated twice?
(d) Now consider the game $G(\infty, \delta)$, i.e. the game $G$ repeated infinitely many times, with discount factor $\delta \in(0,1)$. Define average payoffs as

$$
(1-\delta) \sum_{t=1}^{\infty} \delta^{t} \pi_{i t}
$$

where $\pi_{i t}$ is the stage- $t$ payoff of player $i=1,2$. What does the folk theorem tell us about the set of possible equilibrium payoffs in this game as $\delta$ grows large? Show that there is a Subgame-perfect Nash Equilibrium that achieves 10 as average payoffs for both players.
2. Country $A$ produces oil of a value of $\$ 10$ a year. However, in order to get the oil to the market, country $A$ will have to build a pipeline through either country $B$ or country $C$. Neither country $B$ nor $C$ are oil-producers, but they have to give permission to $A$ in order for the pipeline to be built, and they can demand a payment from $A$ for this. Constructing a pipeline is otherwise costless.
(a) Suppose first that all the oil can be transported by a single pipeline. Let us think of this as a cohesive coalitional game with transferable payoffs. Why can we think of this game as cohesive? Why can we think of it as having transferable payoffs?
(b) Write up the value of the different coalitions and find the core of the coalitional game described in (a). Give an intuition for the outcome.
(c) Now, suppose that (i) $B$ can transport all the oil, but $C$ can at most transport half of it, and (ii) $A$ can transport all the oil via an alternative route, but at a cost of $\$ 5$. Write up the value of the different coalitions and find the core of this coalitional game. How does the outcome differ from the outcome you found in (b)? Give an intuition.
3. In this question, we consider two games in which player 1 takes an action and also chooses whether that action is observable to player 2. These two games are described by Figure 1 and Figure 2, respectively.

The only difference between the games is the order in which player 1 moves: in the first game (Figure 1), player 1 first chooses whether the action is observable, and then takes the action. In the second game (Figure 2), player 1 first takes the action, and then chooses whether the action will be observed.
Notice from the payoffs in the game trees that player 1 must invest 1 in order to make his action observable (i.e. his payoff is lower by 1 whenever he makes the action observable).
(a) Consider the game in Figure 1. Here, player 1 has to decide whether to make his action observable before he takes the action. How many proper subgames are there in this game (not counting the game itself)? What are the strategy sets of each player?
(b) Find all the pure-strategy Subgame-perfect Nash Equilibria of the game in Figure 1.
(c) Consider the equilibrium (equilibria) you found in question (b). If player 1 chooses to reveal his action (plays $I$ ), give an intuition for why this is the case. If player chooses not to reveal his action (plays $N$ ), give an intuition for why this is the case. Make the connection with the idea of commitment.
(d) Now consider the game in Figure 2. Here, player 1 has to decide whether to make his action observable after he takes the action. Argue that Subgame-perfect Nash Equilibrium is not a good solution concept to solve this game. Show that there is a Perfect Bayesian Equilibrium in which the players get payoffs (4,5). Remember to specify full strategies and the beliefs that support the equilibrium.


Figure 1


Figure 2

